Magnetic Lens

Introduction

Scanning electron microscopes image samples by scanning with a high-energy beam of electrons. The subsequent electron interactions produce signals such as secondary and back-scattered electrons than contain information about the sample surface topography. Electromagnetic lenses are used to focus this electron beam down to a spot about 10 nm wide on the sample surface.

Note: This model requires the Particle Tracing Module.

Model Definition

Particles (electrons) are released from near the bottom boundary of the simulation space and pass through a collimator. This collimator can typically be adjusted to remove stray electrons. A simple DC coil produces an axial magnetic field. This rotationally symmetric, inhomogeneous magnetic field results in non-axial electrons experiencing a radial force causing them to spiral about the axis. As they begin to spiral, they have a larger velocity component perpendicular to the mainly axial magnetic field, therefore the radius of their spiral/helical path decreases. Thus, a parallel beam of electrons entering the lens will converge to a point.

If the region in which the magnetic field acts upon the electrons is sufficiently small, this coil acts as a ‘thin’ convex lens and the thin lens expression holds.

Model Equations

A simple model is set up to test the magnetic force within the Particle Tracing for ACDC interface. The equations solved are the equation of motion of a charged particle in a magnetic field (Lorentz force):

\[
\frac{d}{dt}(mv) = q(v \times B)
\]

where \( q \) is the particle charge (C), \( v \) is the particle velocity (m/s) and \( B \) is the magnetic flux density (SI unit: T). The total work done on a particle by a magnetic field is zero.
**Results and Discussion**

The magnetic flux density is plotted in Figure 1. The strength of the lens depends upon the coil configuration and current. The lens within electron microscopes are generally very strong, in some cases focusing the electron beam within the lens itself.

![Multislice: Magnetic flux density norm (T)](image)

**Figure 1: Plot of the magnetic flux density in the magnetic lens.**

**Figure 2** plots the electron trajectories as they travel through the coil. The electrons are focused at a point along the z-axis. The focal length is given by:

\[
\frac{V}{i} = \frac{K}{i^2}
\]

(2)

where \( K \) is a constant based on the coil geometry and number of turns, \( V \) is the accelerating voltage and \( i \) is the coil current. The focal length increases with electron energy (i.e. \( V \)) because their high velocity means they spend less time experiencing a force due the magnetic field. However, as the current increases so does the magnetic
field strength, therefore the electrons spiral in tighter paths bringing the focal length closer.

Figure 2: Plot of the electron trajectories travelling through the magnetic lens.

The ability to change the focal length of a lens is useful as it allows the focusing onto a surface in addition to adjusting the magnification. The effect of the focusing can be seen in Figure 3 which shows a Poincaré map of the particle position at three different snapshots in time. The sharpness of the cross-over can be improved using multiple lenses.
Figure 3: Poincaré plot of the particle location in the xy-plane initially (red), at the focal point of the lens (blue) and at the last time step (black).

Reference


Model Library path: ACDC_Module/Particle_Tracing/magnetic_lens

Modeling Instructions

**MODEL WIZARD**

1. Go to the Model Wizard window.
2. Click Next.
3. In the Add physics tree, select AC/DC>Magnetic Fields (mf).
4 Click Add Selected.
5 Click Next.
6 Find the Studies subsection. In the tree, select Preset Studies>Stationary.
7 Click Finish.

GLOBAL DEFINITIONS

Parameters
1 In the Model Builder window, right-click Global Definitions and choose Parameters.
2 In the Parameters settings window, locate the Parameters section.
3 In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vaccel</td>
<td>500[V]</td>
</tr>
<tr>
<td>Ic</td>
<td>0.32[A]</td>
</tr>
<tr>
<td>velec</td>
<td>sqrt((2<em>Vaccel</em>e_const)/(me_const))</td>
</tr>
<tr>
<td>Nc</td>
<td>1000</td>
</tr>
</tbody>
</table>

GEOMETRY 1
Build a simple coil geometry using cylinders.

1 In the Model Builder window, under Model 1 click Geometry 1.
2 In the Geometry settings window, locate the Units section.
3 From the Length unit list, choose mm.

Cylinder 1
1 Right-click Model 1>Geometry 1 and choose Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 10.
4 In the Height edit field, type 2.5.

Cylinder 2
1 In the Model Builder window, right-click Geometry 1 and choose Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 6.
4 In the Height edit field, type 2.5.
5 Click the Build Selected button.
Cylinder 3
1 In the Model Builder window, under Model 1>Geometry 1 right-click Cylinder 1 and choose Duplicate.
2 In the Cylinder settings window, locate the Position section.
3 In the z edit field, type -7.5.

Cylinder 4
1 In the Model Builder window, right-click Geometry 1 and choose Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 2.
4 In the Height edit field, type 2.5.
5 Locate the Position section. In the z edit field, type -7.5.

Cylinder 5
1 In the Model Builder window, under Model 1>Geometry 1 right-click Cylinder 1 and choose Duplicate.
2 In the Cylinder settings window, locate the Position section.
3 In the z edit field, type -2.5.

Cylinder 6
1 In the Model Builder window, right-click Geometry 1 and choose Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 3.
4 In the Height edit field, type 2.5.
5 Locate the Position section. In the z edit field, type -2.5.

Cylinder 7
1 In the Model Builder window, under Model 1>Geometry 1 right-click Cylinder 1 and choose Duplicate.
2 In the Cylinder settings window, locate the Position section.
3 In the z edit field, type 2.5.

Cylinder 8
1 In the Model Builder window, right-click Geometry 1 and choose Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 3.
4 In the Height edit field, type 2.5.
5 Locate the **Position** section. In the **z** edit field, type **2.5**.

**Cylinder 9**
1 Right-click **Geometry 1** and choose **Cylinder**.
2 In the **Cylinder** settings window, locate the **Size and Shape** section.
3 In the **Radius** edit field, type **20**.
4 In the **Height** edit field, type **50**.
5 Locate the **Position** section. In the **z** edit field, type **-15**.

**Difference 1**
1 Right-click **Geometry 1** and choose **Boolean Operations**>**Difference**.
2 Select the objects **cyl5**, **cyl7**, **cyl3**, and **cyl1** only.
3 In the **Difference** settings window, locate the **Difference** section.
4 Under **Objects to subtract**, click **Activate Selection**.
5 Select the objects **cyl4**, **cyl8**, **cyl2**, and **cyl6** only.
6 Click the **Build Selected** button.
7 Click the **Go to Default 3D View** button on the Graphics toolbar.

**Work Plane 1**
1 Right-click **Geometry 1** and choose **Work Plane**.
2 Click the **Wireframe Rendering** button on the Graphics toolbar.
3 In the **Work Plane** settings window, locate the **Work Plane** section.
4 From the **Plane type** list, choose **Face parallel**.
5 On the object **dif1**, select Boundary 3 only.

**Plane Geometry**
Click the **Zoom Extents** button on the Graphics toolbar.

**Circle 1**
1 In the **Model Builder** window, under **Model 1**>**Geometry 1**>**Work Plane 1** right-click **Plane Geometry** and choose **Circle**.
2 In the **Circle** settings window, locate the **Size and Shape** section.
3 In the **Radius** edit field, type **2**.
4 Click the **Build Selected** button.

Last, create a circular edge to be used in the Multi-Turn Coil Domain feature as a reference edge.
**Work Plane 2**

1. In the **Model Builder** window, right-click **Geometry 1** and choose **Work Plane**.
2. In the **Work Plane** settings window, locate the **Work Plane** section.
3. From the **Plane type** list, choose **Face parallel**.
4. On the object **dif1**, select Boundary 13 only.

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**Circle 1**

1. In the **Model Builder** window, under **Model 1>Geometry 1>Work Plane 2** right-click **Plane Geometry** and choose **Circle**.
2. In the **Circle** settings window, locate the **Object Type** section.
3. From the **Type** list, choose **Curve**.
4. Locate the **Size and Shape** section. In the **Radius** edit field, type 8.
5. Click the **Build All** button.

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**MATERIALS**

Add materials for the air domain and metal collimator and coil.

**Material 1**

1. In the **Model Builder** window, under **Model 1** right-click **Materials** and choose **Material**.
2. In the **Material** settings window, locate the **Material Contents** section.
3 In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Property</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical conductivity</td>
<td>sigma</td>
<td>6e7</td>
</tr>
<tr>
<td>Relative permittivity</td>
<td>epsilonr</td>
<td>1</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>mur</td>
<td>1</td>
</tr>
</tbody>
</table>

**Material 2**
1 In the **Model Builder** window, right-click **Materials** and choose **Material**.
2 Select Domain 1 only.
3 In the **Material** settings window, locate the **Material Contents** section.
4 In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Property</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical conductivity</td>
<td>sigma</td>
<td>0</td>
</tr>
<tr>
<td>Relative permittivity</td>
<td>epsilonr</td>
<td>1</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>mur</td>
<td>1</td>
</tr>
</tbody>
</table>

**MAGNETIC FIELDS**
In the **Model Builder** window, expand the **Model 1>Magnetic Fields** node.

**Multi-Turn Coil Domain 1**
1 Right-click **Magnetic Fields** and choose **Multi-Turn Coil Domain**.
2 Select Domain 4 only.
3 In the **Multi-Turn Coil Domain** settings window, locate the **Coil Type** section.
4 From the list, choose **Circular**.
5 Locate the **Multi-Turn Coil Domain** section. In the **N** edit field, type **Nc**.
6 In the **I coil** edit field, type **Ic**.

Specify the reference edges to be used in the calculation of the current path for the circular coil. To obtain the best results, the selected edges should have a radius close to the average coil radius. In this case, select the edges intentionally created in previous steps.

**Reference Edge 1**
1 Right-click **Model 1>Magnetic Fields>Multi-Turn Coil Domain 1** and choose **Edges>Reference Edge**.
2 In the **Reference Edge** settings window, locate the **Edge Selection** section.
3 Click **Clear Selection**.
4 Select Edges 22, 23, 57, and 82 only.

**Mesh 1**

*Scale 1*
1 In the **Model Builder** window, under **Model 1** right-click **Mesh 1** and choose **Scale**.
2 In the **Scale** settings window, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Select Domains 2–5 only.
5 Locate the **Scale** section. In the **Element size scale** edit field, type 0.5.

*Free Triangular 1*
1 In the **Model Builder** window, right-click **Mesh 1** and choose **More Operations>Free Triangular**.
2 Select Boundary 30 only.

*Size 1*
1 Right-click **Model 1>Mesh 1>Free Triangular 1** and choose **Size**.
2 In the **Size** settings window, locate the **Element Size** section.
3 From the **Predefined** list, choose **Extremely fine**.
4 Click to expand the **Element Size Parameters** section. Locate the **Element Size** section. Click the **Custom** button.
5 Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.
6 In the associated edit field, type 0.4.

*Free Tetrahedral 1*
1 In the **Model Builder** window, right-click **Mesh 1** and choose **Free Tetrahedral**.
2 In the **Settings** window, click **Build All**.

**Study 1**
1 In the **Model Builder** window, expand the **Study 1** node.
2 Right-click **Study 1** and choose **Compute**.
RESULTS

*Magnetic flux density norm*

1. In the Multislice settings window, locate the Multiplane Data section.
2. Find the z-planes subsection. In the Planes edit field, type 0.
3. Click the Plot button.

MODEL 1

In the Model Builder window, right-click Model 1 and choose Add Physics.

MODEL WIZARD

1. Go to the Model Wizard window.
2. In the Add physics tree, select AC/DC>Charged Particle Tracing (cpt).
3. Click Add Selected.
4. Click Next.
5. Find the Studies subsection. In the tree, select Preset Studies for Selected Physics>Time Dependent.
6 Find the Selected physics subsection. In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Physics</th>
<th>Solve for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic Fields (mf)</td>
<td>×</td>
</tr>
</tbody>
</table>

7 Click Finish.

GEOMETRY 1
In the Model Builder window, collapse the Model 1>Geometry 1 node.

DEFINITIONS
In the Model Builder window, collapse the Model 1>Definitions node.

CHARGED PARTICLE TRACING
1 In the Charged Particle Tracing settings window, locate the Domain Selection section.
2 Click Clear Selection.
3 Select Domain 1 only.

Magnetic Force 1
You need to provide the forces acting on the particles; in this case, the magnetic (Lorentz) force.
1 In the Model Builder window, expand the Model 1>Charged Particle Tracing node.
2 Right-click Model 1>Charged Particle Tracing and choose Magnetic Force.
3 Select Domain 1 only.
4 In the Magnetic Force settings window, locate the Magnetic Force section.
5 From the B list, choose Magnetic flux density (mf/mf).

Inlet 1
1 In the Model Builder window, right-click Charged Particle Tracing and choose Inlet.
2 Select Boundary 30 only.
3 In the Inlet settings window, locate the Initial Position section.
4 From the Initial position list, choose Projected plane grid.
5 In the N edit field, type 10000.
6 Locate the Initial Velocity section. In the \( v_0 \) table, enter the following settings:

| velec | z     |
**STUDY 2**

*Step 1: Time Dependent*

1. In the **Model Builder** window, expand the **Study 2** node, then click **Step 1: Time Dependent**.

2. In the **Time Dependent** settings window, click to expand the **Values of Dependent Variables** section.

3. Select the **Values of variables not solved for** check box.

4. From the **Method** list, choose **Solution**.

5. From the **Study** list, choose **Study 1, Stationary**.

6. Locate the **Study Settings** section. Click the **Range** button.

7. Go to the **Range** dialog box.

8. From the **Entry method** list, choose **Number of values**.

9. In the **Stop** edit field, type $5e^{-9}$.

10. In the **Number of values** edit field, type 25.

11. Click the **Replace** button.

12. In the **Model Builder** window, right-click **Study 2** and choose **Compute**.

**RESULTS**

*Particle Trajectories (cpt)*

1. In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node.

2. In the **Model Builder** window, under Results> **Particle Trajectories (cpt)** click **Particle Trajectories 1**.

3. In the **Particle Trajectories** settings window, locate the **Coloring and Style** section.

4. Find the **Line style** subsection. From the **Type** list, choose **Line**.

5. Find the **Point style** subsection. From the **Type** list, choose **None**.

6. In the **Model Builder** window, under Results> **Particle Trajectories (cpt)>>Particle Trajectories 1** click **Color Expression 1**.

7. In the **Color Expression** settings window, locate the **Expression** section.

8. In the **Expression** edit field, type $\sqrt{\text{cpt.Ftx}^2+\text{cpt.Fty}^2+\text{cpt.Ftz}^2}$.
9 Click the Plot button.

Now construct a Poincaré map to visualize the radial distribution of particles initially, at the focal point and at the exit of the modeling domain.

Data Sets
1 In the Model Builder window, expand the Results>Data Sets node.
2 Right-click Data Sets and choose Cut Plane.
3 In the Cut Plane settings window, locate the Plane Data section.
4 From the Plane list, choose xy-planes.
5 In the z-coordinates edit field, type -6.
6 Locate the Data section. From the Data set list, choose Particle 1.
7 Right-click Results>Data Sets>Cut Plane 1 and choose Duplicate.
8 In the Cut Plane settings window, locate the Plane Data section.
9 In the z-coordinates edit field, type 7.
10 Right-click Results>Data Sets>Cut Plane 2 and choose Duplicate.
11 In the Cut Plane settings window, locate the Plane Data section.
12 In the z-coordinates edit field, type 34.
2D Plot Group 3

1. In the Model Builder window, right-click Results and choose 2D Plot Group.
2. Right-click 2D Plot Group 3 and choose More Plots>Poincaré Map.
3. In the Poincaré Map settings window, locate the Data section.
4. From the Cut plane list, choose Cut Plane 3.
5. Locate the Coloring and Style section. From the Color list, choose Black.
6. Click the Zoom Extents button on the Graphics toolbar.
7. Select the Radius scale factor check box.
8. In the associated edit field, type 4E-2.
9. Right-click Results>2D Plot Group 3>Poincaré Map 1 and choose Duplicate.
10. In the Poincaré Map settings window, locate the Data section.
11. From the Cut plane list, choose Cut Plane 1.
12. Locate the Coloring and Style section. In the Radius scale factor edit field, type 2E-2.
13. From the Color list, choose Red.
14. Right-click Results>2D Plot Group 3>Poincaré Map 2 and choose Duplicate.
15. In the Poincaré Map settings window, locate the Data section.
16. From the Cut plane list, choose Cut Plane 2.
17. Locate the Coloring and Style section. From the Color list, choose Blue.
18. In the Radius scale factor edit field, type 5E-3.
19. Click the Plot button.
20 Click the **Zoom Extents** button on the Graphics toolbar.